Appendix: Method for Assigning Shaded Regions in the Graph

Without loss of generality, the method is described using the six-month growth rate of manufacturing industrial production. To use a three-month growth rate, one could do a "find and replace" substituting "400" for "200," "1qtr" for "2qtr," etc.

Step 1: Let $g_q^{GDP-2qtr} = 200\log(\frac{GDP_q}{GDP_{q-2}})$ denote the two-quarter logarithmic growth rate (SAAR) of real GDP in quarter q. Using the 1990q1 – 2014q1 sample, calculate the mean and standard deviation of $g_q^{GDP-2qtr}$ and denote them by $\bar{g}^{GDP-2qtr}$ and $\sigma^{GDP-2qtr}$, respectively. Standardize the growth rates as z-scores:

$$(1) z_q^{GDP-2qtr} = \frac{g_q^{GDP-2qtr} - \bar{g}^{GDP-2qtr}}{\sigma^{GDP-2qtr}}$$

The GDP growth rate thresholds for the shading is the ordered set {-1.5, 0.0, 1.5, 2.5, 3.5}. For example, the second element is 0.0. Denote the *i*th threshold in the set ($1 \leq i \leq 5$) as $t_{pct,i}^{GDP-2qtr}$. Convert it to a logarithmic growth rate:

$$(2) t_{log,i}^{GDP-2qtr} = 100\log(1 + \frac{t_{pct,i}^{GDP-2qtr}}{100})$$

For each of the log thresholds, find its z-score equivalent with equation (1):

$$(3) t_{z,i}^{GDP-2qtr} = \frac{t_{log,i}^{GDP-2qtr} - \bar{g}^{GDP-2qtr}}{\sigma^{GDP-2qtr}}$$

Step 2: Let $g_m^{IPMan-6mth} = 200\log(\frac{IPMan_m}{IPMan_{m-6}})$ denote the six-month logarithmic growth rate (SAAR) of manufacturing IP in month *m*. Using the February 1990–February 2014 sample (chosen to line up with the GDP sample), calculate the mean and standard deviation of $g_m^{IPMan-6mth}$ and denote them by $\bar{g}^{IPMan-6mth}$ and $\sigma^{IPMan-6mth}$, respectively. Standardize the six-month logarithmic growth rates as:

$$(4) z_m^{IPMan-6mth} = \frac{g_m^{IPMan-6mth} - \bar{g}^{IPMan-6mth}}{\sigma^{IPMan-6mth}}$$

Solve for the thresholds for the 6-month logarithmic growth rate of manufacturing IP that are equivalent—in z-score units—to the log GDP growth thresholds in equation (2)

$$(5) t_{log,i}^{IPMan-6mth} = \bar{g}^{IPMan-6mth} + \sigma^{IPMan-6mth} t_{z,i}^{GDP-2qtr}$$

Convert these thresholds from log growth rates to percentage growth rates by

$$(6) t_{pct,i}^{IPMan-6mth} = 100[\exp\left(\frac{t_{log,i}^{IPMan-6mth}}{100}\right) - 1]$$

The collection of thresholds $\{t_{pct,i}^{IPMan-6mth}\}_{i=2}^{5}$ are the boundaries of the shaded regions in the charts. The value $t_{pct,1}^{IPMan-6mth}$ is the truncation threshold; growth rates below this are reassigned this truncation threshold value in the graph. This is done to focus the eye on expansionary periods.