Constructing a “virtual” federal funds rate

Since December 16, 2008, the Federal Open Market Committee (FOMC) has maintained a target range for the federal funds rate of 0 to 0.25 percent. While perhaps not impossible, targeting a federal funds rate below 0 percent would be difficult in practice because of the implicit 0 percent interest rate on currency. Therefore, 0 percent is generally thought of as a lower bound for the federal funds rate; hence the name “zero lower bound.” (See Keister (2011) for a brief discussion of these issues).

However, some economists have argued that monetary policy has been more accommodative since December 2008 than a simple reference to the target federal funds range would indicate. For example, see this excerpt from Chung, LaForte, Reifschneider and Williams’s article “Have We Underestimated the Likelihood and Severity of Zero Lower Bound Events?” in the February 2012 Journal of Money, Credit and Banking (emphasis mine):

“Several recent studies have attempted to estimate the quantitative effect of large scale asset purchases by the Federal Reserve on U.S. long-term interest rates… the general conclusion from this research is that the first phase of the Federal Reserve’s asset purchases reduced the general level of long-term interest rates by around 50 basis points. To put the 50-basis-point figure into perspective, a back-of-the-envelope calculation suggests that achieving such a reduction in long-term yields would require something on the order of a 200- basis-point cut in the federal funds rate. Thus, the overall stimulus provided by the FOMC during 2009 is better characterized as a (virtual) reduction in nominal short-term interest rates to –2%, not to zero.”

Chung et al. (2008) then models the effects of large-scale asset purchases (LSAPs) on the macroeconomy by embedding the portfolio-balance effects of LSAPs on 10-year Treasury yields inside the FRB/US macroeconometric model. The FRB/US simulations indicate that LSAPs beginning in early 2009 and ending with the FOMC’s announced intention “to purchase a further $600 billion of longer-term Treasury securities by the end of the second quarter of 2011” would have the effect of lowering the unemployment rate about 1.5 percentage points by the end of 2012 compared with a baseline without any LSAPs. Simulations of the macroeconomic effects of Federal Reserve purchases of $600 billion in longer-term Treasury securities from an earlier version of the Chung et al paper were also cited in a January 2011 speech titled “The Federal Reserve's Asset Purchase Program” by Federal Reserve Vice Chair Janet Yellen.

Fuhrer and Olivei (2011) find that an immediate and persistent 100 basis point decline in the 10-year Treasury yield raises the level of real gross domestic product (GDP) 2.63 percentage points above baseline after eight quarters in a “largely unstructured” vector autoregression (VAR). These authors find that within the Federal Reserve Bank of Boston’s economic model, the same persistent 100 basis point decline in the 10-year Treasury yield raises the level of real GDP 2.5 percentage points above baseline after eight quarters.

Prior to December 2008, the federal funds rate was commonly thought of as the primary operational tool for monetary policy. Motivated by the findings of Chung et al. (2012) and Fuhrer and Olivei (2011) this essay outlines a method for splicing the federal funds rate prior to January 2009 with a “virtual” federal funds rate from January 2009 to present. As mentioned, January 2009 was the first full month with a 0 to 0.25 percent target fed funds rate. The first step is translating information from the Federal Reserve’s balance sheet into a single indicator of monetary policy.
\[(1) \quad \text{LSAPRatio}_t = 100 \left( \frac{\text{FRSBGW5}_t + \text{FRSBGW6}_t + \text{FRSBMBW}_t}{1000\text{FBLN}_t} \right) \]

where
- **FRSBGW5**: Federal Reserve Banks: U.S. Treasury Security Holdings: Over five to 10 years (EOP, Mil.$). (Haver Analytics ticker FRSBG5W@WEEKLY taken from the Federal Reserve’s H.4.1 release.)
- **FRSBGW6**: Federal Reserve Banks: U.S. Treasury Security Holdings: Over 10 Yrs (EOP, Mil.$). (Haver Analytics ticker FRSBG6W@WEEKLY taken from the Federal Reserve’s H.4.1 release.)
- **FRSBMBW**: Federal Reserve MBS Purchase: Mortgage-Backed Securities (MBS) Held Outright (EOP, Mil.$). (Haver Analytics ticker FRSBBMBW@WEEKLY taken from the Federal Reserve’s H.4.1 release.)
- **FBLN**: Total Liabilities: All Commercial Banks (NSA, Bil.$). (Haver Analytics ticker FBLN@WEEKLY taken from the Federal Reserve’s H.8 release.)

In other words, the LSAP ratio is total Federal Reserve holdings of U.S. Treasuries with five or more years to maturity and MBS scaled by total commercial bank liabilities. This setup is slightly different than Chung et al. (2012). Those authors use all securities with more than one-year until maturity. Chung et al. (2012) use nominal GDP as a scaling factor instead of commercial bank liabilities. (On the other hand, Gertler and Karadi (2011) use total intermediated assets in their model, which is probably more conceptually similar to commercial bank liabilities than nominal GDP.) The Federal Reserve holdings are weekly (Wednesday of each week) while commercial bank liabilities are weekly averages. We assign the commercial bank liabilities to the Wednesday of the week; then we use simple linear interpolation to assign a daily value for all four terms on the right hand side of (1). Finally we take monthly averages of all the (interpolated) daily series on the right hand side of (1) before constructing the ratio. After these conversions, \( \text{LSAPRatio} \) is a monthly series.

The second step is a conversion from the \( \text{LSAPRatio} \) to a virtual federal funds rate. The formula that we use is:

\[ (2) \quad \text{FFR}_t^{\text{virtual}} = d_t \alpha (\text{LSAPRatio}_{\text{Dec2008}} - \text{LSAPRatio}_t) + \text{FFR}_t^{\text{Eff}} \]

where,
- \( \text{FFR}_t^{\text{virtual}} \): The “virtual” federal funds rate whose name is inspired by the above quote from Chung et al. (2012).
- \( d_t \): A dummy variable that is 0 before January 2009 and 1 thereafter.
- \( \text{FFR}_t^{\text{Eff}} \): The monthly average of the effective federal funds rate.
- \( \alpha \): A fixed unknown scalar that will be estimated.

By construction the “virtual” federal funds rate is equal to the effective rate before January 2009. After December 2008, its value depends on \( \alpha \). Clearly if \( \alpha = 0 \), then the “virtual” rate is equal to the effective rate even after December 2008. If, on the other hand, \( \alpha = 0.15 \), say, then in December 2009 the “virtual” federal funds rate would be \(-1.39\%\):

\[ (3) \quad \text{FFR}_{\text{Dec2009}}^{\text{virtual}} = 0.15(1.77\% - 11.86\%) + 0.12\% = -1.39\% \]
We estimate $\alpha$ using a medium to large-size Bayesian vector autoregression (BVAR). The BVAR is reduced form with 13 lags and an estimation sample of 1983m1-2012m01. (I.e., 1983m1-2012m01 data are used to construct the contemporaneous and lagged variables in the variable. Therefore, the date of the first dependent variable observation in the BVAR is 1984m02. We start the sample in 1983m1 because it is roughly thought to coincide with the end of non-borrowed reserves targeting.) The estimation of the BVAR is implemented using the techniques described in Banbura et. al. (2008). The 20 variables included in the BVAR are

<table>
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<tr>
<th>Table 1: Variables in [reduced form] BVAR (1983m1-2012m01)</th>
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<tr>
<td>Unemployment rate (SA) **</td>
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<tr>
<td>Employment/population ratio (SA)**</td>
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<td>ISM Manufacturing: PMI Composite (SA)</td>
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<tr>
<td>Reuters/University of Michigan Consumer Sentiment Index</td>
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<td>Housing starts (SA)</td>
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<td>CoreLogic Home Price Index (SA)**</td>
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<td>CPI-U: All Items (SA)**</td>
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<td>CPI-U: Core (SA) **</td>
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<td>CPI-U: Energy (SA) **</td>
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<td>PPI: Finished Goods (SA) **</td>
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Note: ** indicates log of variable is used.
Sources: Haver Analytics; Bureau of Labor Statistics; Census Bureau; Federal Reserve Board; Financial Times; Wall Street Journal; Commodity Research Bureau; Institute for Supply Management; CoreLogic; Reuters/University of Michigan

Where possible, we use data that has fairly small revisions. For example, revisions to the unemployment rate are generally due to changes in seasonal adjustment factors. Real GDP is not in the model because its revisions can be quite large; for example fourth quarter 2008 growth was revised from $-3.8$ percent in the “advance” (first) release to $-6.3$ percent in the “final” (third) release to $-8.9$ percent in the current vintage release (as of March 2012). We want to use series with only minor revisions to incorporate the belief that the FOMC cannot react to revisions it has not seen. One exception to this rule are the series on home prices and housing starts; since the housing sector is thought a particularly important sector for the monetary policy transmission mechanism, the benefit of including these series seemed to outweigh the cost.

Because the BVAR has 20 variables, Bayesian shrinkage is needed to prevent over-fitting of the VAR. Using the same notation as in Banbura et. al. (2008) we set the overall shrinkage parameter $\lambda = 0.05$. ($\lambda = 0$ implies the posterior equals the prior while $\lambda = \infty$ implies the posterior coincides with the OLS estimate of the VAR.) As in Banbura et al. (2008), $\lambda$ is chosen so that the RMSE of the forecasts of three key variables in the BVAR (the unemployment rate, Core CPI, and the “virtual” fed funds rate) is (about) the same as the root-mean-square error (RMSE) for a three variable VAR with only the three key variables. Each variable in the BVAR
has an associated $\delta_i$ parameter used to construct the prior as in Banbura et al. (2008). Variables thought to have substantial mean reversion use $\delta_i = 0$ while other variables (corresponding to a random walk prior) have $\delta_i = 1$. We set $\delta_i = 0$ for the unemployment rate, the ISM Manufacturing Index and the Reuters/University of Michigan Index of Consumer Sentiment. For all of the other variables in the BVAR we set $\delta_i = 1$. We include the “sum of coefficients” prior described in Banbura et al. (2008) using $\tau = 10 \lambda$ as in the paper. Let $y_t(\alpha)$ denote the $1 \times 20$ row-vector of time $t$ data. The last entry of $y_t(\alpha)$ is the “virtual” federal funds, which depends on $\alpha$ (after December 2008). Let $x_t(\alpha)$ denote the first 13 lags of $y_t(\alpha)$ stacked in a row vector with the constant 1 appearing at the end; i.e. $x_t(\alpha) = [y_{t-1}(\alpha) \ y_{t-2}(\alpha) \ldots y_{t-13}(\alpha) \ 1]$. The data matrices for the VAR without incorporating the prior information are $X(\alpha) = [x_1(\alpha)' \ x_2(\alpha)' \ldots x_{T-1}(\alpha)' \ x_T(\alpha)']'$ and $Y(\alpha) = [y_1(\alpha)' \ y_2(\alpha)' \ldots y_{T-1}(\alpha)' \ y_T(\alpha)']'$ where $T$ is the number of observations in the VAR. Incorporating prior information into the estimation can be achieved by augmenting the data matrices with $T_d$ dummy observations $X_d$ and $Y_d$; the augmented matrices are then $X_*(\alpha) = [X(\alpha)' \ X_d']'$ and $Y_*(\alpha) = [Y(\alpha)' \ Y_d']'$. The posterior maximum likelihood estimates (MLE) of the BVAR regression coefficients are:

\[
(4) \overline{B}(\alpha) = (X_*(\alpha)X_*(\alpha))^{-1}X_*(\alpha)'Y_*(\alpha)
\]

The MLE estimate of the covariance matrix of the residuals is:

\[
(5) \overline{\Sigma}(\alpha) = \frac{1}{T + T_d} (Y_*(\alpha) - X_*(\alpha)\overline{B}(\alpha))(Y_*(\alpha) - X_*(\alpha)\overline{B}(\alpha))'
\]

The log likelihood of the BVAR (where $n = 20$ is the number of variables in the system) is [see Hamilton (1994) page 293]

\[
(6) \mathcal{L}(\alpha, \overline{B}(\alpha), \overline{\Sigma}(\alpha))
= -\frac{(T + T_d)n}{2} \ln(2\pi) + \frac{T + T_d}{2} \ln|\overline{\Sigma}(\alpha)|^{-1}
- \frac{1}{2} \sum_{t=1}^{T + T_d} (y_t^*(\alpha) - x_t^*(\alpha)\overline{B}(\alpha))\overline{\Sigma}(\alpha)^{-1}(y_t^*(\alpha) - x_t^*(\alpha)\overline{B}(\alpha))'
\]

In (6) $y_t^*(\alpha)$ is a row of $Y_*(\alpha)$ and $x_t^*(\alpha)$ is a row of $X_*(\alpha)$. The MLE estimate of $\alpha$ is

\[
(7) \hat{\alpha} = \arg\max_{\alpha} \mathcal{L}(\alpha, \overline{B}(\alpha), \overline{\Sigma}(\alpha))
\]

In words, for any candidate value of $\alpha$, first construct the “virtual” federal funds rate using (2). Then, construct the (dummy-augmented) data matrices $X_*(\alpha)$ and $Y_*(\alpha)$. Next, compute the MLE estimates of the BVAR regression coefficients and the covariance matrix of the residuals using (4) and (5). Finally, compute the posterior likelihood with (6). The value of $\alpha$ that maximizes the left hand side of (6) is the MLE estimate. In our application it turns out to be $\hat{\alpha} = 0.1494$. The plot of the “virtual” federal funds rate for this value of $\alpha$ is shown here:
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References: